Monitoring Massive Network Traffic using Bayesian Inference

> David Rodriguez Cisco Systems, Inc. Senior Research Engineer

> > November 7, 2018

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

Team

Dhia Mahjoub Scott Sitar Gilad Ranier Matt Foley Irwin Fule-Ver Skyler Hawthorne Thomas Matthew

Table: We are the research-engineering team implementing algorithms and maintaining the DNS threat intelligence to the Cisco Umbrella product.

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

Table of contents

Observe Signals

Generate Signals

Fit Fast

Fit Many

Fit Over Time

Measure Risk

Plan

Observe Signals

Generate Signals

Fit Fast

Fit Many

Fit Over Time

Measure Risk

Signals of Threats

Heuristic Fallout



Figure: The combinatorial explosion of query patterns highlight patterns with *zero* queries. Also, notice, some patterns are similar if permuted.

Detecting anomalies associated with threats are hard to determine ${\rm if}^1$:

- the domain has previous query volume
- there is large variations in query volume
- there are gaps between periods with query volume



Observe Signals

Generate Signals

Fit Fast

Fit Many

Fit Over Time

Measure Risk

Be the Adversary

Question

What if roles were reversed? Rather than observing, you were asked to generate *malicious* traffic.

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへで

Be the Adversary

Question

What if roles were reversed? Rather than observing, you were asked to generate *malicious* traffic.

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

You might need some tools, but that's not a problem.

Common Discrete Distributions

Observation

If you can generate a random number then you can definitely generate any one of these:

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

- Geom(p) the geometric
- $Pois(\lambda)$ the poisson
- Bin(n, p) the binomial
- NB(n, p) the negative binomial

Common Discrete Distributions²



Figure: Clockwise starting top left: geometric, poisson, negative binomial, and binomial distributions. For given parameters 100 samples generated per distribution.

²likely not seen in the real traffic

Common Discrete Distributions ³



Figure: Example query volume to **jd.com** over the last 30 days is bimodal and therefore not one of the previous distributions.

ж

イロト イポト イヨト イヨト

³likely not seen in the real traffic

Mixtures of Discrete Distributions

We can *mix* distributions.⁴

Zero Inflated Distributions

$$f(x;\theta) = \psi \mathcal{I}_0 + (1-\psi)g(x;\theta)$$
(1)

where \mathcal{I}_0 is an indicator variable at zero, $\psi \in [0, 1]$, and $g(x; \theta)$ is any discrete distribution from the previous slide.

⁴be careful to maintain the properties of a probability distribution $\langle \cdot \rangle$ \Rightarrow $\langle \cdot \rangle \sim \langle \cdot \rangle$

Spam Filtering as Mixtures of Distributions⁵



Figure: Other applications using mixtures of distributions are spam filters where *spam* and *ham* can be seen a web *topics*. Certain words appear more frequently within topics. [2]

⁵Think of an equation like this: $f(x) = \sum_{i}^{n} \psi_{i} f_{i}(x)$ where $\sum_{i} \psi_{i} = 1$ is a solution

Zero Inflated Simulations

Puzzle

Pick an urn with probability p. If you pick urn A draw 0. If you pick urn B draw a number from a negative binomial distribution. Start over.

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 臣 のへぐ

Zero Inflated Simulations



Figure: Picking a zero with probability p otherwise picking a number from a negative binomial.

▲ロト ▲帰 ト ▲ ヨ ト ▲ ヨ ト ・ ヨ ・ の Q ()

24 Hour Simulations





Figure: Zero-Inflated Poissons (*Zip*) with $\psi = .30$ along with $\lambda = 5, 10, 20, 30$

Figure: Zero-Inflated Negative Binomials (*Zinb*): $\psi = .3, n = 10, p = .01, .3, .4, .6$

(日)、

э

Admittedly, our little game has limitations.

Puzzle

Consider hourly counts from one day to known botnets, phishing, dns-tunneling. Suppose, the order of the hours don't matter, can we simulate daily traffic with a $Zinb(\psi, p, n)$?⁶

⁶for some ψ , p, n that we can choose.

Simulating Malicious Traffic⁷



Figure: Botnet domain a1a79b359237e.hosting with *Zinb*(0.13, 0.45, 3.24)



Figure: Phishing domain support-globomail.com with *Zinb*(0.50, 0.25, 2.01)

⁷Images on left *real* the right *simulated*

Simulating Malicious Traffic⁸



Figure: Phishing domain universal-ads.com with Zinb(0.83, 0.39, 9.07)



Figure: Phishing domain clientes-moopixel.com with *Zinb*(0.10, 0.41, 17.81)

⁸Image on left *real* the right *simulated*

Simulation Fit

Note

Be skeptical, just because a simulation looked good once, it might have been rare.

・ロト・日本・モト・モート ヨー うへで

Measure of Fit to Malicious Traffic





Figure: a1a79b359237e.hosting

Figure: support-globomail.com





Figure: universal-ads.com

Figure: clientes-moopixel.com

Figure: QQ-Plots where tighter bands provide evidence the simulated data agrees with the observed. Wider bands, show more uncertainty.

Plan

Observe Signals

Generate Signals

Fit Fast

Fit Many

Fit Over Time

Measure Risk



Rainier supported by Stripe, Inc.⁹ and authored by Avi Bryant¹⁰ is an open-source Bayesian Inference project written in Scala. The appeal of this project is:

- functional API with higher order function abstractions
- efficient hierarchical model fitting for datasets fitting in memory
- community of collaborators working on problems related to predictive modeling and risk and fraud detection

Bayesian Inference and Monte Carlo Simulations



Figure: Bayesian inference is iterative process of drawing samples from *priors* (sometimes accepting and rejecting the sample) then updating a *posterior* distribution. There are variety of *sampling* algorithms: *Gibb*, *No U*-*Turn* (*NUTS*), *Leap Frog*, etc.

 $Example \ Bayesian \ Sampling [1] \ {}_{\mbox{via Gibbs sampling}}$

Bayesian Sampling with data-augmentation

1: procedure GIBBS SAMPLER \triangleright Estimating ψ and θ $\psi^{(0)} \leftarrow \mu_0$ 2: \triangleright $u_0 \sim Uniform(0,1)$ 3: $\theta^{(0)} \leftarrow \theta_{n}$ \triangleright random θ_0 4: for $t \leftarrow 1, \ldots$ do Generate $z_{i}^{(t)}$ (i = 1, ..., n) from (j = 1, ..., k) 5: $P(z_i^{(t)} = j | \psi_i^{(t-1)}, \theta_i^{(t-1)}, x_i) \propto \psi_i^{(t-1)} f(x_i | \theta_i^{(t-1)})$ 6: Generate $\psi^{(t)}$ from $\pi(\psi|z^{(t)})$ 7: Generate $\theta^{(t)}$ from $\pi(\theta|z^{(t)},x)$ 8: end for Q٠ return $\psi^{(n)}, \theta^{(n)}$ 10: 11: end procedure

Sampling from Mixtures



Figure: Two $Zinb(\psi, p, n)$ where the parameters ψ, p, n have different prior distributions. Some priors are considered *non-informative* and should be handled carefully.

▲ロト ▲帰 ト ▲ ヨ ト ▲ ヨ ト ・ ヨ ・ の Q ()

Hello Rainier

Listing 1: Fitting Zero Inflated Negative Binomial in Rainier

```
import com.stripe.rainier.core. {NegativeBinomial, LogNormal, Beta }
 1
 2
      import com.stripe.rainier.sampler.{RNG, ScalaRNG}
 3
 4
      case class Zinb(psi; Double, p; Double, n; Double)
 5
 6
      object ZinbMCMC extends Serializable {
 7
        implicit val rng: RNG = ScalaRNG(1527608515939L)
 8
 9
        def fit(data: Seq[Int]): Zinb = {
10
          val priors = for \{
            p < - Beta(2, 5), param
11
12
            n < -LogNormal(0, 1).param
13
          } yield (p, n)
14
15
          val psi = for \{
16
            (p, n) < - priors
            psi < - Beta(2, 2).param
17
18
            fit < - NegativeBinomial(p, n).zeroInflated(psi).fit(data)
19
          } yield psi
20
21
          // ... vour decide
22
          // ... call priors.sample() or psi.sample() for sequence of values
23
24
          Zinb(fitPsi, fitP, fitN)
25
26
```

Plan

Observe Signals

Generate Signals

Fit Fast

Fit Many

Fit Over Time

Measure Risk

Massive Parallelization

Trick

Using Apache Spark we can distribute our simulations and run as many as we would like in parallel. $^{11}\,$

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへぐ

¹¹http://spark.apache.org

Massive Parallelization



Figure: Passing chunks of the file(s) into rdd partitions, in Spark, distributes the Rainier simulations.

▲□▶ ▲圖▶ ★ 国▶ ★ 国▶ - 国 - のへで

Puzzle

Given a file where each row contains a **(domain, day, Seq[Int])** write a program using Rainier to fit a zero inflated negative binomial distribution.

Hello Spark and Rainier ¹²

Listing 2: Dispatching the Zinb simulation (to days worth simulating).

```
1
      trait Event {
 2
        val name: String
3
4
5
6
        val time: String
      case class Dormant(name: String, time: String) extends Event
 7
      case class Singleton(name: String, time: String, value: Int) extends Event
 8
      case class MultiState(name: String, time: String, values: Seq[Int]) extends Event
 9
10
      def zinbDispatcher(event: Event): Zinb = {
11
         event match {
12
          case Dormant(_, _) => Zinb(0.0, 0.0, 0.0)
13
          case Singleton(, , value) => Zinb(1/2.40, 1/2, value*2)
          case MultiState(_, _, values) => ZinbMCMC.fit(values)
14
15
16
```

¹²Completing the example: sc.textFile(pathToFile).map(assignState).map(zinbDispatcher) \circ Q (

Gotcha

Common errors occur with serialization of the rainier simulations. The previous example, not by accident, wrapped the *Zinb* simulation in a Serializable object. Another possibility, is to use:

com.twitter.chill.Meatlocker(f)

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

chill is shipped with Spark.

Plan

Observe Signals

Generate Signals

Fit Fast

Fit Many

Fit Over Time

Measure Risk

Scheduling the Processing

Major challenges in deciding:

How many minutes/hours/days should be fit.

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

• How long between *fitting* each signal.

Scheduling Windows



Figure: Some simulations can be run at non-overlapping intervals, overlapping intervals, and varied time windows.

▲ロト ▲帰ト ▲ヨト ▲ヨト 三日 - の々ぐ

Notes on Aggregation and Disaggregation

Note

The idea of *aggreagation* over a *large* window of time that is subsequently compared to an aggregation over a *small* window of time has been studied in problems related to **itermittant** demand. [4]

Plan

Observe Signals

Generate Signals

Fit Fast

Fit Many

Fit Over Time

Measure Risk



Measuring Risk

Goal

Exploit the parameterization of the fitted models to define a statistical measure of *rarity*.

Examples of common statistical tests:

- Given a data point x_i and a probability distribution f(x; θ) compute the p-value.
- Given data points: x₁,..., x_n and two models
 f(x; θ₁), g(x; θ₂) compute the likelihood that the points are from one distribution rather than another.

Two Risk Measures



Figure: Parameters fit to previous observations of a signal can be used to analyze new observations in batch or streaming ways.

イロト 不得 トイヨト イヨト

э

Risk Scores

Observation

We also want to *accumulate* risk-measures over time where more recent events contribute *more* to the score than older events. We can do this using exponential moving averages.

Given a new risk measure X_i at time *i* then update a time-dependent risk score *S* as follow:

$$S_i = X_i w + (1 - w) S_{i-1}$$
 (2)

with $w \in [0, 1]$.

Time-Dependent Risk Measures



 Φ is normal cdf

Figure: Example of trending historical μ_i, σ_i where more recent values contribute more.

Sample Pipeline



Cyclical Trends Storage

Figure: Example data pipeline where the most recent simulations are input to a historical database containing previous fitted parameters. Then, finally, a risk-score job fires off by reconciling the historical with the most recent simulation updating a chosen risk score.

Risk All Wrong¹³



Figure: How not to create a risk score. Here the the risk-score per parameter is trended per weekday causing inappropriate correlations

¹³Additionally, there are good reasons why not to trend the parameters of a model.

DIEBOLT, J., AND ROBERT, C. P. Estimation of finite mixture distributions through bayesian sampling.

Journal of the Royal Statistical Society. Series B (Methodological) 56, 2 (1994), 363-375.

Jordan, M. I.

Graphical models.

Statistical Science 19, 1 (2004), 140–155.

LAMBERT, D., AND LIU, C.

Adaptive thresholds.

Journal of the American Statistical Association 101, 473 (Mar 2006), 78–88.

NIKOLOPOULOS, K., SYNTETOS, A. A., BOYLAN, J. E., Petropoulos, F., and Assimakopoulos, V. An aggregate-disaggregate intermittent demand approach (adida) to forecasting: an empirical proposition and analysis. Journal of the Operational Research Society 62, 3 (Mar 2011), 544-554 ◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへぐ Discrete Pobability Distribution Updates: Adaptive Thresholding

Puzzle (Lambert, et al [3])

Update a negative binomial distribution NB(p, n) from a stream of counts: x_1, x_2, x_3, \ldots

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

Adaptive Thresholding

The **trick** is that not all values should contribute to updating the underlying parameters to NB(p, n). In other words, outliers should be corrected or handled *robustly*.



Adaptive Thresholding



Two points worth exploring in the methods we've discussed are:

- ▶ Updating the distributions *NB*(*p_i*, *n_i*) over time
- Tracking outlier significance