Design and Implementation of the GraphBLAS Template Library (GBTL)

Scott McMillan, Samantha Misurda

Marcin Zalewski, Peter Zhang, Andrew Lumsdaine

Software Engineering Institute
Carnegie Mellon University
Pittsburgh, PA 15213

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DM-0003820

What is this talk about?

- GraphBLAS
  - an emerging paradigm for graph computation
  - programs new graph algorithms in a highly abstract *language of linear algebra*.
  - executes in a wide variety of programming environments
- Our implementation of GraphBLAS
  - Graph BLAS Template Library (GBTL)
  - High-level C++ *frontend* (some features still in progress)
  - Algorithms written in terms of the API
  - Released at: https://github.com/cmu-sei/gbtl

Graph BLAS Forum
http://www.graphblas.org

Software Engineering Institute
Carnegie Mellon University

CREST
Indiana University

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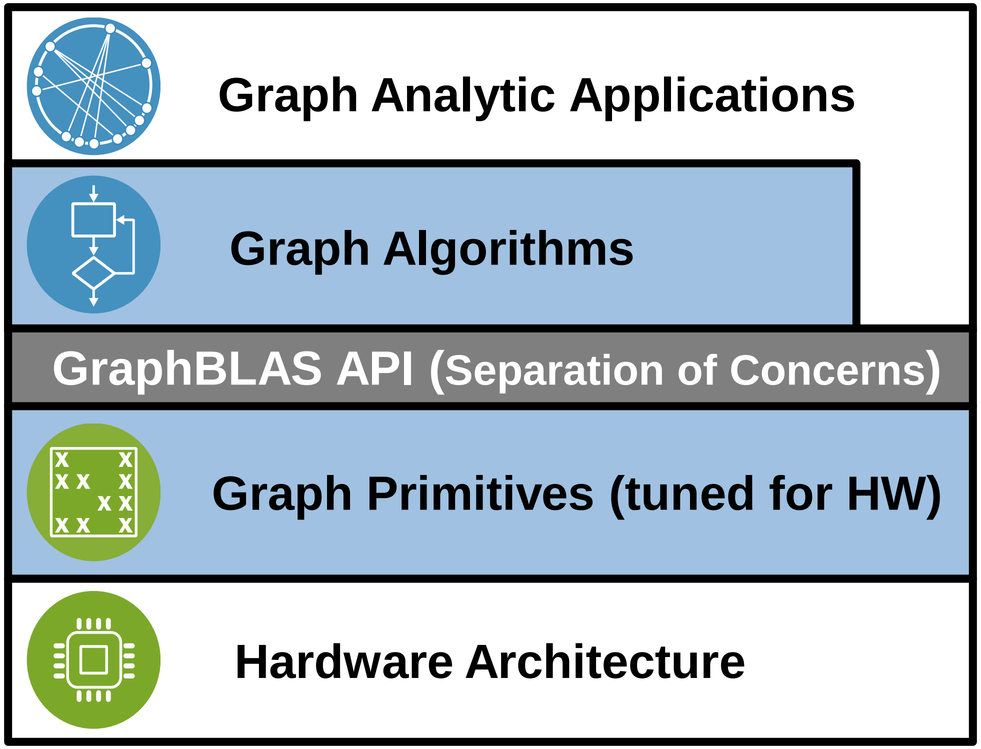
Design Goals

- Separation of concerns:
  - Algorithm development
  - Hardware tuning
- Low overhead
- Support patterns for scalable, high-performance computing



- Templated C++ implementation
  - Allows for generic programming and template metaprogramming
  - Allows generic semantic checks (compile time)
- Similarity to the C API specification (under development)
- Easy to use

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- **Operations Overview**
- Object Design
- Operations – Function Signatures
- Example Algorithm
- Summary and Future Work

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Background: GraphBLAS Operations

Operation	Description	Old Name
MxM, MxV, VxM	Perform matrix *multiplication* (e.g., BFS traversal)	SpGEMM
EwiseAdd, EwiseMult	Element-wise *addition* and *multiplication* of matrices (e.g., graph union, intersection)	SpEWiseX
Extract	Extract a sub-matrix from a larger matrix (e.g., sub-graph selection)	SpRef
Assign	Assign to a sub-matrix of a larger matrix (e.g., sub-graph assignment)	SpAsgn
Apply	Apply *unary function* to each element of matrix (e.g., edge weight modification)	Apply
Reduce	*Reduce* along columns or rows of matrices (e.g., vertex degree)	Reduce
Transpose	Swaps the rows and columns of a matrix (e.g., reverse directed edges)	Transpose
BuildMatrix	Build a matrix from row, column, value tuples	Sparse
ExtractTuples	Extract the row, column, value tuples from a matrix	Find

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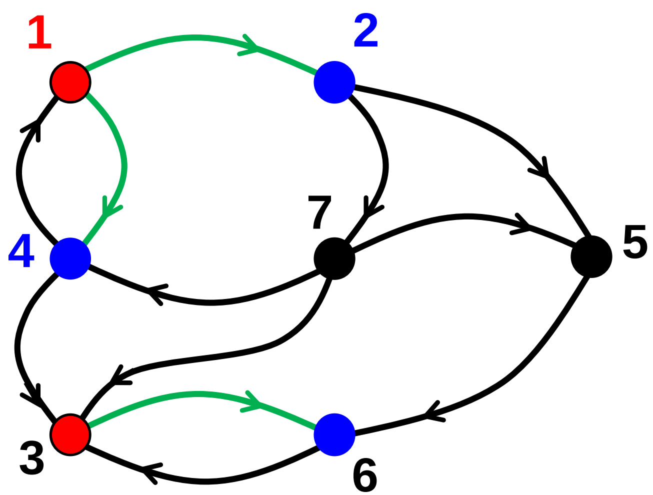
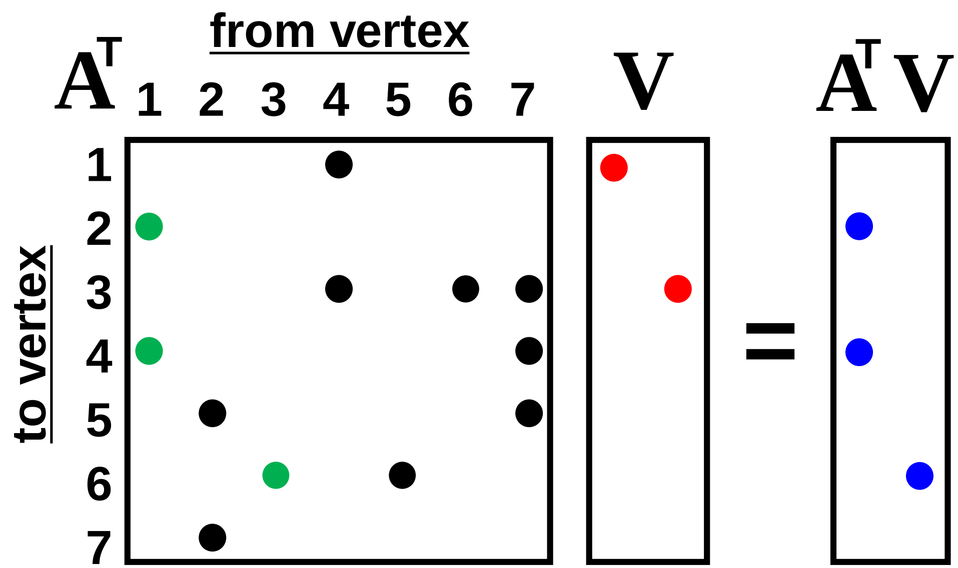
Background: Matrix Multiply (MxM)

$$C = A \oplus . \otimes B$$





- The **Semiring** ($\oplus . \otimes$) determines how this computation is carried out.
- Consists of two **Monoids** (**Binary Function** + identity)
  - $\otimes$, e.g., (multiply, 1)
  - $\oplus$, e.g., (add, 0)
- These can be user defined, not adhering strictly to Semiring properties

$$c_{i,j} = \sum_{l=1}^{k} a_{i,l} \times b_{l,j}$$

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GraphBLAS Operation: MxM example

$$\mathbf{C} = \mathbf{A} \oplus . \otimes \mathbf{B}$$

- Required:
  - Two input matrices: A_{MxK} and B_{KxN}
  - One output matrix: C_{MxN}
  - One semiring: $\oplus . \otimes$

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GraphBLAS Operation: MxM example

$$C \oplus= A^T \oplus.\otimes B^T$$

- Required:
  - Two input matrices: A^T_{MxK} and B^T_{KxN}
  - One output matrix: C_{MxN}
  - One semiring: $\oplus.\otimes$
- Optional:
  - Matrix Transpose – only necessary on inputs
  - Accumulation – binary function, can be different from Semiring's $\oplus$.

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GraphBLAS Operation: MxM example

$$\mathbf{C}(\neg\mathbf{M}) \oplus= \mathbf{A}^{\mathbf{T}} \oplus.\otimes\ \mathbf{B}^{\mathbf{T}}$$

- Required:
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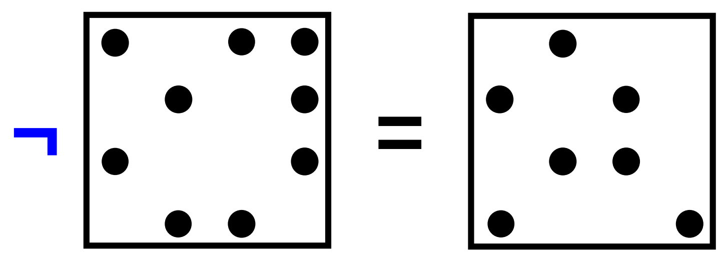
  - One output matrix: C_{MxN}
  - One semiring: $\oplus.\otimes$
- Optional:
  - Matrix Transpose – only necessary on inputs
  - Accumulation – binary function, can be different from Semiring's $\oplus$.
  - Output Mask: M_{MxN} – specifies which locations in C can be modified
  - Mask Complement
    - Invert the structure of stored values (sparse)
    - Invert boolean values (dense)



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blue – optional parameters
red – optional modifiers

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GBTL Operations

Operation	Math	Out	Inputs
MxM	$\mathbf{C}(\neg\mathbf{M}) \oplus= \mathbf{A}^{\mathbf{T}} \oplus.\otimes \mathbf{B}^{\mathbf{T}}$	$\mathbf{C}$	$\neg$, $\mathbf{M}$, $\oplus$, $\mathbf{A}$, $\mathbf{T}$, $\oplus.\otimes$, $\mathbf{B}$, $\mathbf{T}$
MxV (VxM)	$\mathbf{c}(\neg\mathbf{m}) \oplus= \mathbf{A}^{\mathbf{T}} \oplus.\otimes \mathbf{b}$	$\mathbf{c}$	$\neg$, $\mathbf{m}$, $\oplus$, $\mathbf{A}$, $\mathbf{T}$, $\oplus.\otimes$, $\mathbf{b}$
EwiseMult	$\mathbf{C}(\neg\mathbf{M}) \oplus= \mathbf{A}^{\mathbf{T}} \otimes \mathbf{B}^{\mathbf{T}}$	$\mathbf{C}$	$\neg$, $\mathbf{M}$, $\oplus$, $\mathbf{A}$, $\mathbf{T}$, $\otimes$, $\mathbf{B}$, $\mathbf{T}$
EwiseAdd	$\mathbf{C}(\neg\mathbf{M}) \oplus= \mathbf{A}^{\mathbf{T}} \oplus \mathbf{B}^{\mathbf{T}}$	$\mathbf{C}$	$\neg$, $\mathbf{M}$, $\oplus$, $\mathbf{A}$, $\mathbf{T}$, $\oplus$, $\mathbf{B}$, $\mathbf{T}$
Reduce (row)	$\mathbf{c}(\neg\mathbf{m}) \oplus= \oplus_j \mathbf{A}^{\mathbf{T}}(:,j)$	$\mathbf{c}$	$\neg$, $\mathbf{m}$, $\oplus$, $\mathbf{A}$, $\mathbf{T}$, $\oplus$
Apply	$\mathbf{C}(\neg\mathbf{M}) \oplus= f(\mathbf{A}^{\mathbf{T}})$	$\mathbf{C}$	$\neg$, $\mathbf{M}$, $\oplus$, $\mathbf{A}$, $\mathbf{T}$, f
Transpose	$\mathbf{C}(\neg\mathbf{M}) \oplus= \mathbf{A}^{\mathbf{T}}$	$\mathbf{C}$	$\neg$, $\mathbf{M}$, $\oplus$, $\mathbf{A}$ $(\mathbf{T})$
Extract	$\mathbf{C}(\neg\mathbf{M}) \oplus= \mathbf{A}^{\mathbf{T}}(\mathbf{i},\mathbf{j})$	$\mathbf{C}$	$\neg$, $\mathbf{M}$, $\oplus$, $\mathbf{A}$, $\mathbf{T}$, $\mathbf{i}$, $\mathbf{j}$
Assign	$\mathbf{C}(\neg\mathbf{M})\ (\mathbf{i},\mathbf{j}) \oplus= \mathbf{A}^{\mathbf{T}}$	$\mathbf{C}$	$\neg$, $\mathbf{M}$, $\oplus$, $\mathbf{A}$, $\mathbf{T}$, $\mathbf{i}$, $\mathbf{j}$
BuildMatrix	$\mathbf{C}(\neg\mathbf{M}) \oplus= \mathbb{S}^{mxn}(\mathbf{i},\mathbf{j},\mathbf{v},\oplus)$	$\mathbf{C}$	$\neg$, $\mathbf{M}$, $\oplus$, $\oplus$, m, n, $\mathbf{i}$, $\mathbf{j}$, $\mathbf{v}$
ExtractTuples	$(\mathbf{i},\mathbf{j},\mathbf{v}) = \mathbf{A}(\neg\mathbf{M})$	$\mathbf{i},\mathbf{j},\mathbf{v}$	$\neg$, $\mathbf{M}$, $\mathbf{A}$

Notation: **i,j** – index arrays, **v** – scalar array, **m** – 1D mask, **other bold-lower** – vector (column),
M – 2D mask, **other bold-caps** – matrix, **T** – transpose, **¬** - structural complement,
$\oplus$ monoid/binary function, $\oplus.\otimes$ semiring,
blue – optional parameters, **red** – optional modifiers

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Objects

- Index and IndexArray
- Matrices and vectors
  - Structure and values
  - Sparse (and dense, but not today)
- Modifiers
  - Structural Complement (and Masks)
  - Transpose
- Mathematical operations
  - Binary functions vs. Monoids
  - Semirings
  - Accumulation (just another binary function)

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Indices

- Index – a value used to locate a position in vectors or matrices (pair of indices).
- IndexArray – multiple values used to locate multiple positions

```
// Some typedefs to give GraphBLAS names to some concepts
typedef uint64_t                    IndexType;
typedef std::vector<IndexType>  IndexArrayType;
```


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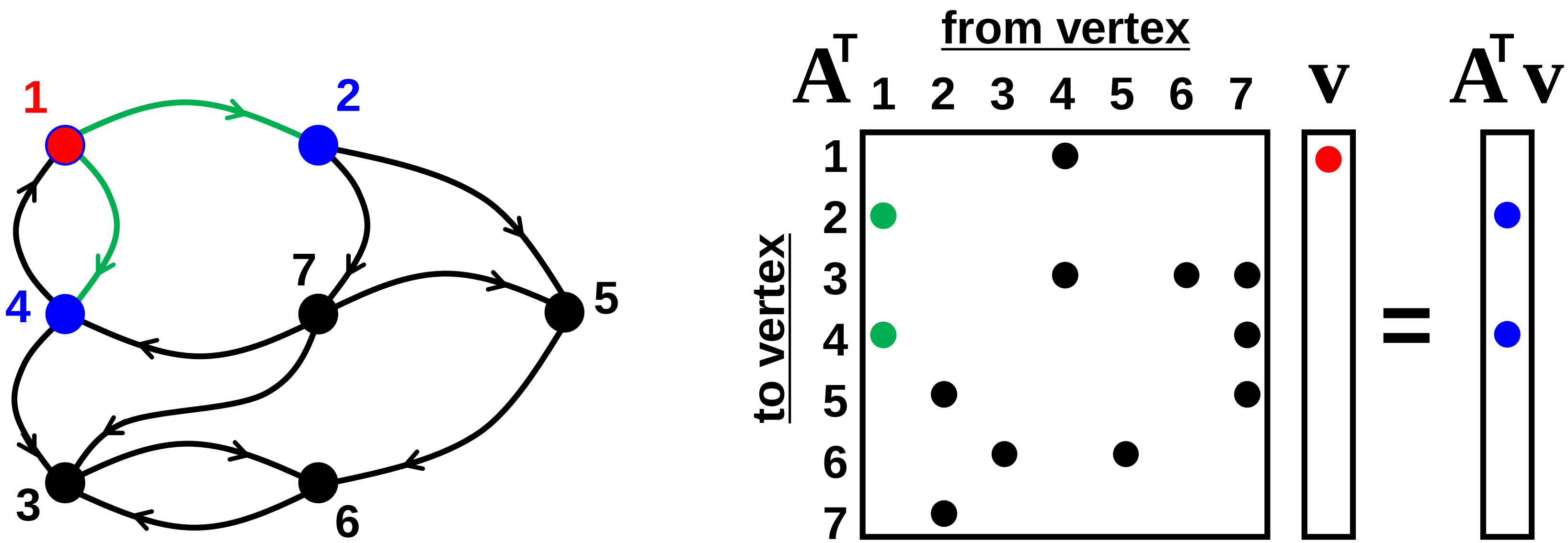
Sparse Matrices and Vectors



Matrices (e.g. adjacency or incidence)

- Adjacency matrices:
  - “Stored values” – edge
  - “Structural zeros” – no edge/storage
- Two index arrays for positions (structure)
- Scalar array for edge attribute (values)

Vectors (e.g. wavefronts)

- Current: (m x 1) or (1 x n) matrices
- In progress: a different object
  - More efficient storage managing only one index dimension
  - Is an implicit orientation necessary (column in mathematics)?

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Achieving Opaque Matrices

- Frontend: Interface here, type/storage details are opaque

```
// Variadic template parameters provide hints for backend matrix type
template <typename ScalarT, typename... TagsT>
class Matrix
{
public:
    // construct an empty matrix with immutable dimensions
    Matrix(IndexType num_rows, IndexType num_cols);

    // Interface, forwards calls to backend
    IndexType get_nnz() const    { return m_matrix.get_nnz(); }
    IndexType get_nrows() const  { return m_matrix.get_nrows(); }
    IndexType get_ncols() const  { return m_matrix.get_ncols(); }
    //...

private:
    detail::matrix_type_generator::result<     // template metaprogramming
        ScalarT,                               // to select backend type
        detail::SparsenessCategoryTag,         // at compile time.
        detail::DirectednessCategoryTag,
        TagsT...>::type
            m_mat;

    friend void template<...> mxm(...);      // all frontend ops are friends
};
```


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Achieving Opaque Matrices (what the user writes)

- Frontend `Matrix` construction:
  - User provides hints (tags) through template parameter packing,
  - backend can make decisions based on hints.

```
Matrix<double, DenseMatrixTag, DirectedMatrixTag>
    matrix(num_rows, num_columns);
```

- “Sparseness” and “directedness” hints currently implemented.
  - Future support could include layout hints like “fast-column access”
- Backend `Matrix` classes:
  - Specialized for hardware and implementation
  - Does not have to support or adhere to hints

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Modifiers

- For two cases
  - Mask complement
  - Matrix transpose
- Purpose:
  - Reduce the number of parameters in operation signatures.
  - Improve readability/usability
- Lightweight wrappers for certain input arguments
- Non-mutating
  - Does not proscribe creation of temporaries

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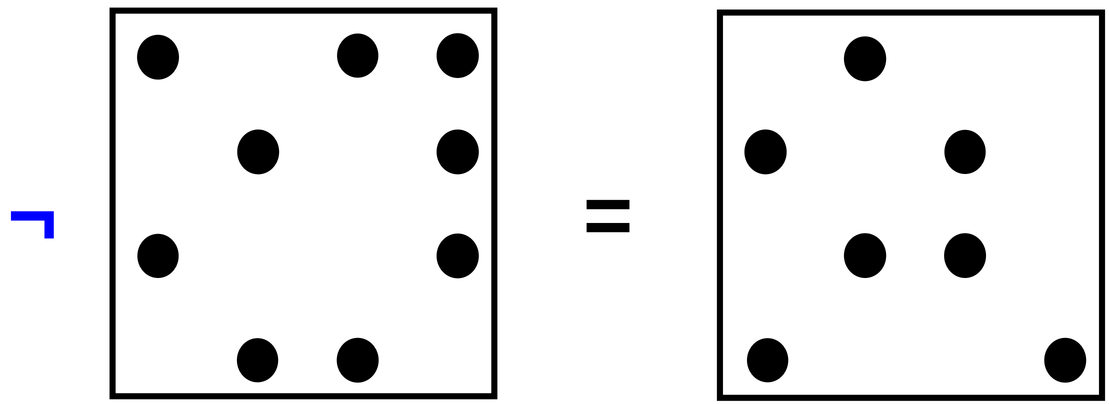
Masks and Structural Complement (WIP)



- Current:
  - Masks also implemented as matrices
  - Values = don’t care
  - Complement changes structure (structural zeros become “1”)
- Future: a different object
  - Potential: more efficient storage with no scalar values

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Complement Modifier (WIP)

- Currently called NegateView (bad name! – will change)
- Templated operations will accept either Matrix or Mask.

```
template <typename MaskT>
class NegateView
{
public:
    NegateView(MaskT const &mask) : m_mask(mask) {}
    IndexType get_nnz() const {
        return m_mask.get_nrows()*m_mask.get_ncols() - m_mask.get_nnz(); }
    //...
private:
    MaskT const &m_mask;
};
```

- Corresponding operation returns a wrapped (*backend*) mask.

```
template<typename MaskT>
NegateView<MaskT> negate(MaskT const &m) {
    return NegateView<MaskT>(backend::negate(m.m_mat));
}
```


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Transpose Modifier

- Effectively swaps row and column access.

```
template <typename MatrixT>
class TransposeView
{
public:
    TransposeView(MatrixT const &matrix) : m_matrix(matrix) {}

    IndexType get_nrows() const { return m_matrix.get_ncols(); }
    IndexType get_ncols() const { return m_matrix.get_nrows(); }
    // ...
private:
    MatrixT const &m_matrix;
};
```

- Corresponding operation returns a wrapped (*backend*) matrix:

```
template<typename MatrixT>
TransposeView<MatrixT> transpose(MatrixT const &A) {
    return TransposeView<MatrixT>(backend::transpose(A.mat));
}
```


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Mathematical Operations: $\oplus$, $\otimes$, $\oplus$=, $\oplus.\otimes$

- Binary functions on multiple domains: D1 x D2 → D3

```
template <typename ResultT, typename Arg1T=ResultT, typename Arg2T=ResultT>
struct ArithmeticMultiplyFunc
{
    ResultT operator()(Arg1T const &lhs, Arg2T const &rhs) const {
        return static_cast<ResultT>(lhs) * static_cast<ResultT>(rhs); }
};
```


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Mathematical Operations: $\oplus$, $\otimes$, $\oplus$=, $\oplus.\otimes$

- Binary functions on multiple domains: D1 x D2 → D3
- Monoids (if needed), include the identity for the operation

```
template <typename ResultT, typename Arg1T=ResultT, typename Arg2T=ResultT>
struct ArithmeticAddMonoid
{
    ResultT operator()(Arg1T const &lhs, Arg2T const &rhs) const {
        return static_cast<ResultT>(lhs) + static_cast<ResultT>(rhs);

    ResultT identity() { return static_cast<ResultT>(0) };
};
```


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Mathematical Operations: $\oplus$, $\otimes$, $\oplus$=, $\oplus.\otimes$

- Binary functions on multiple domains: D1 x D2 → D3
- Monoids (if needed), include the identity for the operation
- Semirings for matrix multiply
  - Two binary functions (and identities, if needed)
  - Addition monoid is defined on one domain: D3 x D3 → D3

```
template <typename ResultT, typename Arg1T=ResultT, typename Arg2T=ResultT>
struct ArithmeticSemiring
{
    // Additive Monoid
    ResultT zero() const { return static_cast<ResultT>(0); }
    ResultT add(ResultT const &lhs, ResultT const &rhs) const {
        return (lhs + rhs); }

    // Multiplicative Monoid
    ResultT one() const { return static_cast<ResultT>(1); }
    ResultT mult(Arg1T const &lhs, Arg2T const &rhs) const {
        return static_cast<ResultT>(lhs) * static_cast<ResultT>(rhs); }
};
```


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Accumulation: ⊕, ⊗, ⊕=, ⊕.⊗

- Select one of two binary functions

```
// No accumulation (select rhs), used as the default parameter value
template <typename ResultT>
struct Assign
{
    ResultT operator()(ResultT lhs, ResultT rhs) { return rhs; }
};



// Accumulation: using arithmetic addition as the default
template <typename ResultT,
          typename BinaryFuncT = ArithmeticAddFunc<ResultT> >
struct Accum
{
    ResultT operator()(ResultT lhs, ResultT rhs) {
               return BinaryFuncT()(lhs, rhs);
    }
};
```


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GBTL Signatures: MxM

$$C(\neg M) \oplus= A^T \oplus.\otimes B^T$$

```
template<typename AMatrixT,
         typename BMatrixT,
         typename CMatrixT,
         typename MaskT,
         typename SemiringT,
         typename AccumT    = math::Assign<typename CMatrixT::ScalarType> >
void mxmMasked(AMatrixT const &A,
               BMatrixT const &B,
               CMatrixT       &C,
               MaskT    const &M,
               SemiringT       sr,
               AccumT          accum = AccumT())
{
    same_dimension_check(C, M, std::string(“mxmMasked"));
    multiply_dimension_check(A, B, C, std::string("mxmMasked"));
    backend::mxmMasked(A.m_mat, B.m_mat, C.m_mat, M.m_mat, sr, accum);
}
```


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GBTL Signatures: MxM

Modifiers appear at the call site.

$$C(\neg M) \oplus= A^{T} \oplus.\otimes B^{T}$$

```
template<typename AMatrixT,
         typename BMatrixT,
         typename CMatrixT,
         typename MaskT,
         typename SemiringT,
         typename AccumT    = math::Assign<typename CMatrixT::ScalarType> >
void mxmMasked(AMatrixT const &A,
               BMatrixT const &B,
               CMatrixT       &C,
               MaskT    const &M,
               SemiringT       sr,
               AccumT          accum = AccumT())
{
    same_dimension_check(C, M, std::string(“mxmMasked"));
    multiply_dimension_check(A, B, C, std::string("mxmMasked"));
    backend::mxmMasked(A.m_mat, B.m_mat, C.m_mat, M.m_mat, sr, accum);
}
```


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$$C(\neg M) \oplus= A^T \oplus.\otimes B^T$$

```
template<typename AMatrixT,
         typename BMatrixT,
         typename CMatrixT,
         typename MaskT,
         typename SemiringT,
         typename AccumT    = math::Assign<typename CMatrixT::ScalarType> >
void mxmMasked(AMatrixT const &A,
               BMatrixT const &B,
               CMatrixT       &C,
               MaskT    const &M,
               SemiringT       sr,
               AccumT          accum = AccumT())
{
    same_dimension_check(C, M, std::string(“mxmMasked"));
    multiply_dimension_check(A, B, C, std::string("mxmMasked"));
    backend::mxmMasked(A.m_mat, B.m_mat, C.m_mat, M.m_mat, sr, accum);
}
```

Simple code like dimension checks in the frontend

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$$C(\neg M) \oplus= A^T \oplus.\otimes B^T$$

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template<typename AMatrixT,
         typename BMatrixT,
         typename CMatrixT,
         typename MaskT,
         typename SemiringT,
         typename AccumT    = math::Assign<typename CMatrixT::ScalarType> >
void mxmMasked(AMatrixT const &A,
               BMatrixT const &B,
               CMatrixT       &C,
               MaskT    const &M,
               SemiringT       sr,
               AccumT          accum = AccumT())
{
    same_dimension_check(C, M, std::string(“mxmMasked"));
    multiply_dimension_check(A, B, C, std::string("mxmMasked"));
    backend::mxmMasked(A.m_mat, B.m_mat, C.m_mat, M.m_mat, sr, accum);
}
```

Forward work
to the backend

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GBTL Signatures: EwiseMult

$$C(\neg M) \oplus= A^T \otimes B^T$$

```
template<typename AMatrixT,
         typename BMatrixT,
         typename CMatrixT,
         typename MaskT,
         typename BinaryFunctionT,
         typename AccumT = Assign<typename CMatrixT::ScalarType> >
void ewiseaddMasked(AMatrixT const  &A,
                    BMatrixT const  &B,
                    CMatrixT        &C,
                    MaskT    const  &M,
                    BinaryFunctionT  func = BinaryFunctionT(),
                    AccumT           accum = AccumT())
{
    same_dimension_check(A, B, std::string("ewiseadd"));
    same_dimension_check(A, C, std::string("ewiseadd"));
    same_dimension_check(C, M, std::string("ewiseadd"));
    backend::ewiseadd(A.m_mat, B.m_mat, C.m_mat, M.m_mat, func, accum);
}
```


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GBTL Signatures: Apply

$$C(\neg M) \oplus= f(A^T)$$

```
template<typename AMatrixT,
         typename CMatrixT,
         typename MaskT,
         typename UnaryFunctionT,
         typename AccumT = Assign<typename CMatrixT::ScalarType> >
void applyMasked(AMatrixT const  &A,
                 CMatrixT        &C,
                 MaskT           &M,
                 UnaryFunctionT   func,
                 AccumT           accum = AccumT())
{
    same_dimension_check(A, C, std::string("apply"));
    same_dimension_check(C, M, std::string("apply"));
    backend::applyMasked(a.m_mat, c.m_mat, M.m_mat, func, accum);
}
```


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GBTL Signatures: Reduce (rows)

$$c(\neg m) \oplus= \oplus_j A^T(:,j)$$

```
template<typename AMatrixT,
         typename CVectorT,
         typename MaskT,
         typename MonoidT = ArithmeticAddMonoid<typename AMatrixT::ScalarType,
                                               typename BMatrixT::ScalarType,
                                               typename CMatrixT::ScalarType >,
         typename AccumT = Assign<typename CMatrixT::ScalarType> >
void rowReduceMasked(AMatrixT const &A,
                     CVectorT       &c,
                     MaskT          &m,
                     MonoidT         monoid = MonoidT(),
                     AccumT          accum = AccumT())
{
    same_dimension_check(c, m, std::string(“rowReduceMasked"));
    if (A.get_nrows() != c.get_nrows() || c.get_ncols() != 1) {
        throw graphblas::DimensionException("rowReduceMasked dimension error");
    }
    backend::rowReduceMasked(A.m_mat, c.m_mat, m.m_mat, moniod, accum);
}
```


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GBTL Signatures: Extract

$$C(\neg M) \oplus= A^T(i,j)$$

```
template<typename AMatrixT,
         typename CMatrixT,
         typename MaskT,
         typename AccumT = Assign<typename CMatrixT::ScalarType> >
void extractMasked(AMatrixT       const &A,
                   IndexArrayType const &i,
                   IndexArrayType const &j,
                   CMatrixT             &C,
                   MaskT                &M,
                   AccumT                accum = AccumT())
{
    same_dimension_check(C, M, std::string("extract"));
    assign_extract_dimension_check(A, C, i.begin(),j.begin());
    backend::extract(A.m_mat, i, j, C.m_mat, M.m_mat, accum);
}
```


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Contents

- Operations Overview
- Object Design
- Operations – Function Signatures
- **Example Algorithm**
- Summary and Future Work

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GBTL Algorithm: Multi-front, Level BFS v0

```
template <typename MatrixT>              // MatrixT scalar type: Integer,..
void bfs_level(MatrixT const &graph,     // MxM adjacency matrix {0,1}
               MatrixT        wavefront, // MxR columns init'd with roots {0,1}
               MatrixT       &levels)    // MxR level results for each BFS
{
   using T = typename MatrixT::ScalarType;

   IndexType rows = wavefront.get_nrows();
   IndexType cols = wavefront.get_ncols();
   MatrixT not_visited(rows, cols);
   T current_level = 0;

   while (wavefront.get_nnz() > 0) {
      // Increment and apply current level to all newly visited nodes.
      arithmetic_n<T, Times<T> > apply_level(++current_level);
      apply(wavefront, levels, apply_level, Accum<T>());

      mxm(wavefront, graph, wavefront,
          IntBooleanSemiring<T>());

      // Remove previously visited vertices from the wavefront
      apply(levels, not_visited, IsZero<T>());
      ewisemult(not_visited, wavefront, wavefront, AndFunc<T>());
   }
}
```

$++c$

$\mathbf{L} += c\,\mathbf{W}$

$\mathbf{W} = \mathbf{W} \,|.\&\, \mathbf{A}$

$\overline{\mathbf{V}} = (\mathbf{L} == 0)$

$\mathbf{W} = \overline{\mathbf{V}} \,.\&\, \mathbf{W}$

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GBTL Algorithm: Multi-front, Level BFS v1

```
template <typename MatrixT>                // MatrixT scalar type: Integer,..
void bfs_level(MatrixT const &graph,       // MxM adjacency matrix {0,1}
               MatrixT        wavefront,   // MxR columns init'd with roots {0,1}
               MatrixT       &levels)      // MxR level results for each BFS
{
   using T = typename MatrixT::ScalarType;

   IndexType rows = wavefront.get_nrows();
   IndexType cols = wavefront.get_ncols();
   MatrixT not_visited(rows, cols);
   T current_level = 0;

   while (wavefront.get_nnz() > 0) {
      // Increment and apply current level to all newly visited nodes.
      arithmetic_n<T, Times<T> > apply_level(++current_level);
      apply(wavefront, levels, apply_level, Accum<T>());

      mxm(transpose(graph), wavefront, wavefront,
          IntBooleanSemiring<T>());

      // Remove previously visited vertices from the wavefront
      apply(levels, not_visited, IsZero<T>());
      ewisemult(not_visited, wavefront, wavefront, AndFunc<T>());
   }
}
```

$++c$

$\mathbf{L} += c\,\mathbf{W}$

$\mathbf{W} = \mathbf{A}^{\mathrm{T}} \,|.\&\, \mathbf{W}$

$\overline{\mathbf{V}} = (\mathbf{L} == 0)$

$\mathbf{W} = \overline{\mathbf{V}} \,.\&\, \mathbf{W}$

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GBTL Algorithm: Multi-front, Level BFS v2

```
template <typename MatrixT>                    // MatrixT scalar type: Integer,..
void bfs_level(MatrixT const &graph,           // MxM adjacency matrix {0,1}
               MatrixT        wavefront,       // MxR columns init'd with roots {0,1}
               MatrixT       &levels)          // MxR level results for each BFS
{
   using T = typename MatrixT::ScalarType;

   IndexType rows = wavefront.get_nrows();
   IndexType cols = wavefront.get_ncols();
   MatrixT not_visited(rows, cols);
   T current_level = 0;

   while (wavefront.get_nnz() > 0) {
      // Increment and apply current level to all newly visited nodes.
      arithmetic_n<T, Times<T> > apply_level(++current_level);
      apply(wavefront, levels, apply_level, Accum<T>());

      mxmMasked(transpose(graph), wavefront, wavefront,
                negate(levels), IntBooleanSemiring<T>());

      // Remove previously visited vertices from the wavefront
      apply(levels, not_visited, IsZero<T>());
      ewisemult(not_visited, wavefront, wavefront, AndFunc<T>());
   }
}
```

$$W(\neg L) = A^T \,|.\&\, W$$

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GBTL Algorithms: Maximal Independent Set

```
template <typename MatrixT>
void mis(MatrixT const &graph,              // NxN
         MatrixT       &independent_set,    // Nx1: !0 indicates node ‘in’ set.
         double         seed = 0)
{
    graphblas::IndexType rows, cols, r, c;
    graph.get_shape(rows, cols);
    // check dimensions...

    generator.seed(seed);  // for SetRandom functor (not shown)

    typedef Matrix<double, /*Tags...*/> RealMatrix;
    using T = typename MatrixT::ScalarType;

    // This will hold the set (non-zero) implies part of the set
    RealMatrix neighbor_max(rows, 1);
    RealMatrix new_members(rows, 1);
    RealMatrix new_neighbors(rows, 1);
    RealMatrix prob(rows, 1);

    RealMatrix candidates(rows, 1);  fill(candidates, 1.0);
    RealMatrix degrees(rows, 1);     fill(degrees, 1.0);

    // Compute degree of each node, add 1 to prevent divide by zero
    rowReduce(graph, degrees, ArithmeticAddMonoid<double, T, T>());
```


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GBTL Algorithms: Maximal Independent Set

```
    while (candidates.get_nnz() > 0)
    {
        // Assign random values (scaled by degree) to all non-zero candidate elements.
        // Ensures that any ties that occur between neighbors will eventually be
        //   broken, and higher degree nodes are more likely selected.
        ewisemult(candidates, degrees, prob, SetRandom());

        // find the neighbor of each source node with the max random number
        mxm(graph, prob, neighbor_max, MaxSelect2ndSemiring<double>());

        // Select source node if its probability is > neighbor_max
        ewiseadd(prob, neighbor_max, new_members, GreaterThan<double>());

        // Add new members to independent set.
        ewiseadd(independent_set, new_members, independent_set, OrFn<double>());

        // Zero out candidates of new_members selected for independent set
        ewisemult(negate(new_members), candidates, candidates);

        if (candidates.get_nnz() == 0) { break; }   // Early exit

        // Neighbors of new members can also be removed
        mxm(graph, new_members, new_neighbors, MaxSelect2ndSemiring<double>());

        // Zero out candidates of new member neighbors
        ewisemult(negate(new_neighbors), candidates, candidates);
    }
}
```


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Summary and Future Work

- GraphBLAS Template Library
  - Separation of concerns: hardware tuning vs. algorithm design
  - C++ templates
    - Expressive syntax in algorithm development (we could do more).
    - Low overhead (first session)
  - Similar to C API Specification (where possible/reasonable)

- Current and Future Work
  - Tracking C API Specification decisions
    - Multiple domains in monoids and semirings
    - Mask and structural complement
    - Variants of basic operations (reduce to scalar, assign/extract columns)
  - More algorithms
  - GraphBLAS C++ API Specification and Reference Implementation

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Questions?

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BACKUPS

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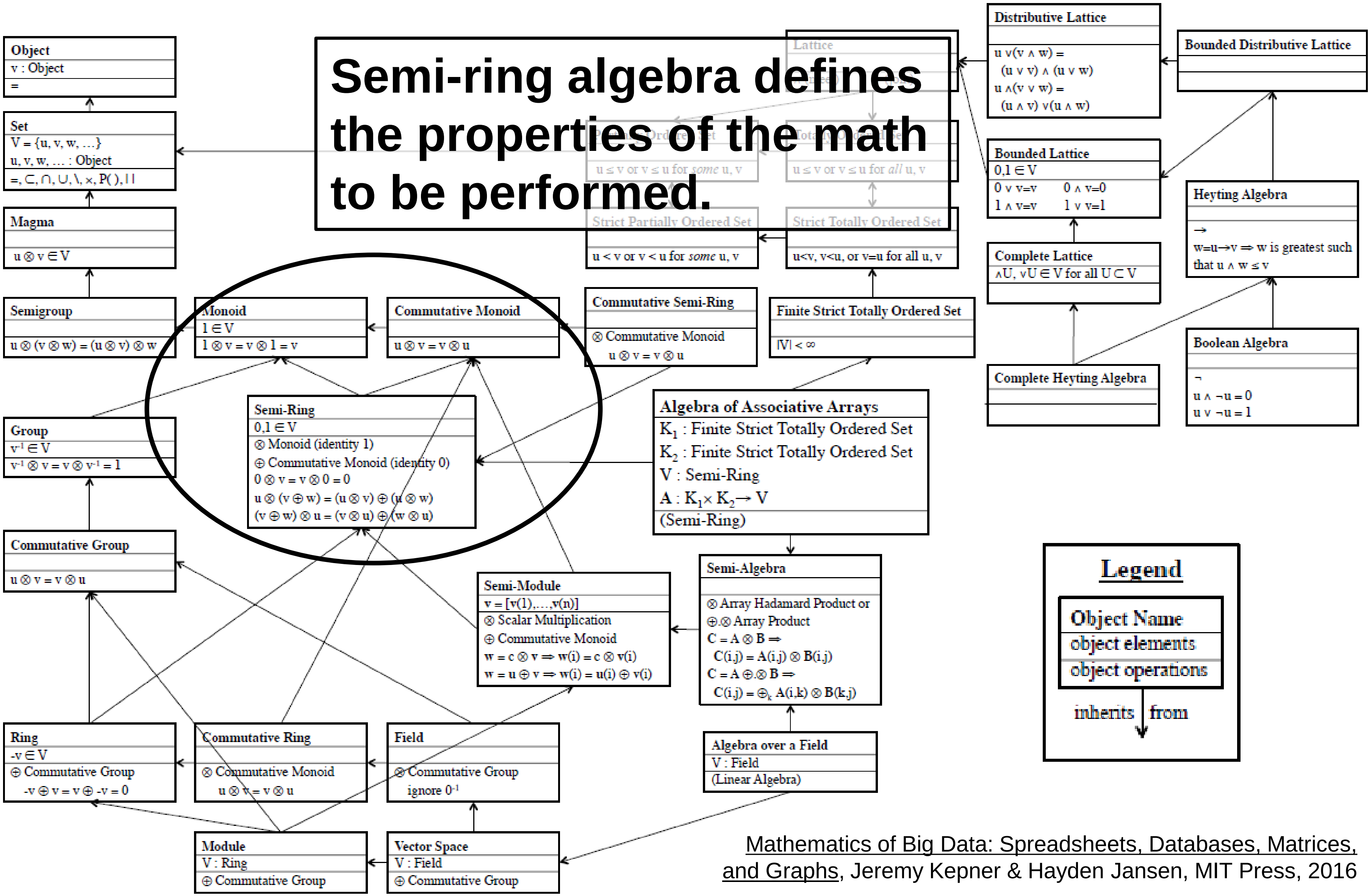
Background: Mathematics of Big Data

Semi-ring algebra defines the properties of the math to be performed.



Mathematics of Big Data: Spreadsheets, Databases, Matrices, and Graphs, Jeremy Kepner & Hayden Jansen, MIT Press, 2016

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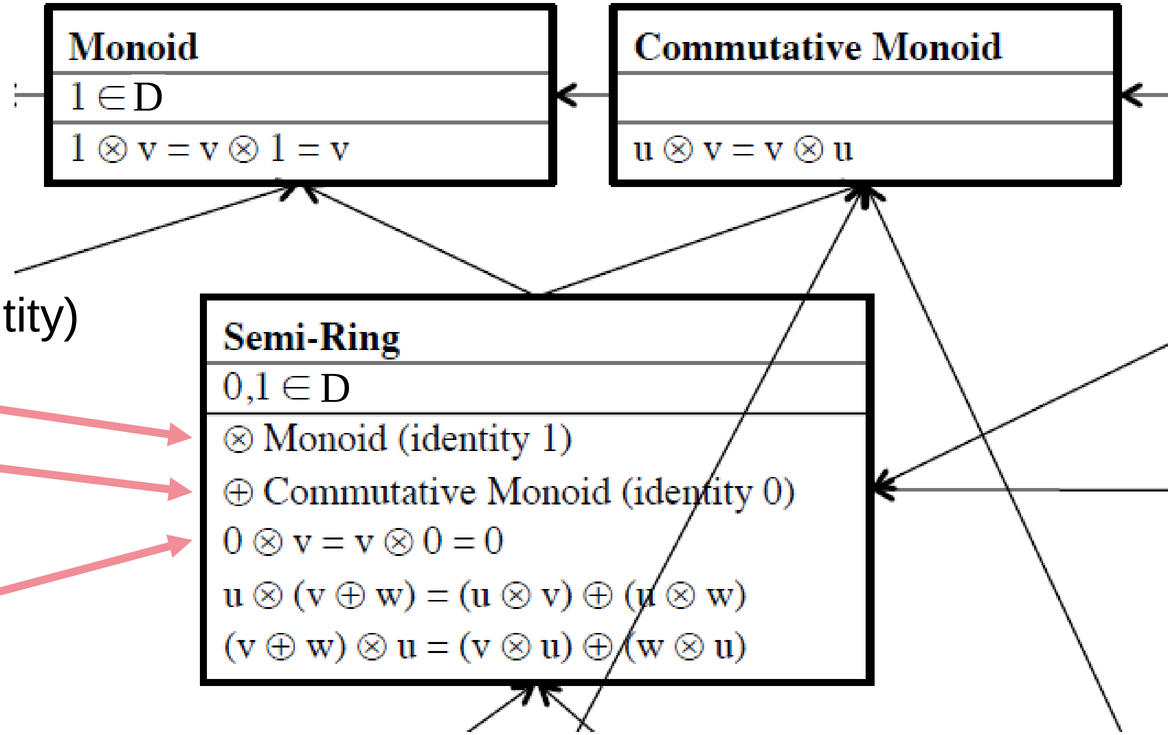
Background: Semi-Ring, Monoid, Binary Function



One Domain, D

Two Monoids (binary func, identity)

- $\otimes$, ("multiply", "1")
- $\oplus$, ("add", "0")

Additive identity
= Multiplicative annihilator
= "Structural zero"

However…

- User can specify arbitrary binary function pairs (still refer to as $\otimes$ and $\oplus$)
- Operating over multiple domains
- That may not have a multiplicative annihilator

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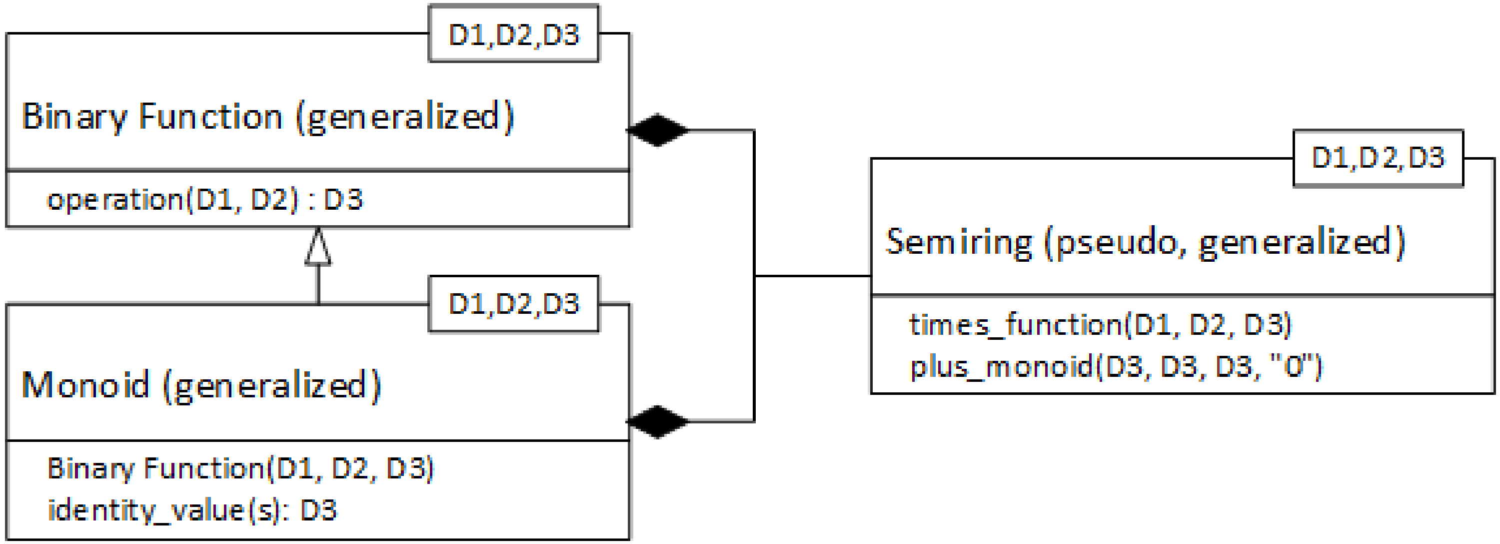
"Generalized" Semiring Design

- Binary multiply function on multiple domains: D1 x D2 → D3
- Additive monoid on one domain: D3 x D3 → D3
- Additive identity specified on D3
- Current discussion:
  - Do we need a "Generalized Monoid" shown here?
  - Do we need to specify the multiplicative identity?
  - Do we require overlapping domains for the additive identity: $0 \in D_1 \cap D_2 \cap D_3$



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Transpose Operation

Overloaded for 3 semantics:

1. Return a TransposeView wrapper around a *backend* Matrix.

```
// Modifier
template<typename MatrixT>
TransposeView<MatrixT> transpose(MatrixT const &A) {
    return TransposeView<MatrixT>(backend::transpose(A.mat));
}
```

2. Populate a new matrix with the scalar values transposed.
3. Modify the internal storage of existing matrix

```
// GraphBLAS operation: can check if A and C are the same object,
// and transpose in place if so.
template<typename MatrixT>
void transpose(MatrixT const &A, MatrixT &C) {...}
```


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